APPLICATION OF OPERATING WORK INDICES TO EVALUATE INDIVIDUAL SECTIONS IN AUTOGONOUS-SEMIAUTOGONOUS/BALL MILL CIRCUITS

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ABSTRACT

Optimum use of power in grinding, both in terms of grinding efficiency and use of installed capital, can have a large effect on profitability. Mill operators have long used operating work indices to evaluate grinding circuit efficiency, with great utility. Operating work indices can be used to evaluate individual units in a conventional circuit or the overall performance of an autogenous-semiautogenous (AG-SAG)/ball mill circuit, but they are of limited value in evaluating AG-SAG and ball mill sections separately. This problem arises due to the difficulty of describing particle size distributions with different shapes by one number, the eighty percent passing size. This paper discusses the operating factors influencing the transfer size distribution and proposes a methodology for evaluating the efficiency of each section of a grinding circuit using the operating work index.

INTRODUCTION

The correlation between the power applied in a grinding operation and the amount of size reduction performed has been intuitively known and several relationships between these two factors have been proposed. F.C. Bond’s third theory has been shown to provide the best correlation in decades of power-based design and analyses of conventional grinding circuits (Bond 1961). Bond’s third theory is presented in [1]. By rearranging the formula to [2], the operating work index (Wio) of an existing grinding circuit can be calculated, given known mill power draw, throughput, feed and product size distributions.

\[
\text{kWh/t} = \frac{\text{Power applied per ton of feed to the grinding unit}}{\text{Wi} = \text{The standard Bond work index of the ore}}
\]

\[
W_{io} = \left( \frac{10}{\sqrt{P_{80}} - \sqrt{F_{80}}} \right)
\]

kWh/t = Power applied per ton of feed to the grinding unit
Wi = The standard Bond work index of the ore
P80 = The 80% cumulative passing size of the product in microns
F80 = The 80% cumulative passing size of the feed in microns

In the 1970s, Rowland proposed using the Wio as a method of analysing plant data, and later addressed its use in assessing milling efficiency (1998). For efficiently operated circuits, the calculated Wio should be close to the appropriate measured work index. Many users apply standard correction factors before comparing the Wio to measured work indices (Bond 1961, Rowland 1978).

The operating work index concept can be applied overall to AG/SAG circuits, but can be misleading when applied to individual sections (the SAG section or ball mill section). As an example, Table 1 presents AG/SAG pilot plant data for an oxide gold ore. Although the overall Wio is close to the BWi, the calculated figures would suggest that the AG/SAG mill is very inefficient, while the ball mill is significantly more efficient than Bond would predict. Figure 1 presents the size distribution data graphically; it is readily evident from Figure 1 that the AG/SAG circuit (transfer) product has a much flatter slope than that of the final product.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed Size, F80 (mm)</td>
<td>177</td>
</tr>
<tr>
<td>Transfer Size, T80 (μm)</td>
<td>726</td>
</tr>
<tr>
<td>Product Size, P80 (μm)</td>
<td>74</td>
</tr>
<tr>
<td>AG/SAG Wio, AWio (kWh/t)</td>
<td>28.5</td>
</tr>
<tr>
<td>Ball Mill Wio, BWio</td>
<td>8.6</td>
</tr>
<tr>
<td>Circuit Wio, OWio</td>
<td>14.6</td>
</tr>
<tr>
<td>Bond Ball Mill Work Index, BWi, kWh/t</td>
<td>13.4</td>
</tr>
<tr>
<td>Bond Rod Mill Work Index, RWi, kWh/t</td>
<td>18.1</td>
</tr>
</tbody>
</table>

Table 1. Example of AG/SAG Ball Mill Circuit Wio Calculations
The apparent difference between ball mill and AG/SAG mill efficiency is an artefact of Bond's implicit assumption that the particle size distributions can be defined by one point, the 80% passing size. In fact, Bond noted that this convenient assumption was valid only in cases where the cumulative particle size distributions were roughly parallel in log-log space (1950). MacPherson (1989) discussed his findings that in most AG/SAG circuits, the parallel line assumption is incorrect. He indicated that an AG/SAG circuit product typically contains much more fines than a stage crushed (or rod milled) product having an equivalent 80% passing size; the dashed line in Figure 1 illustrates this effect. As a result, operating work indices calculated for AG/SAG sections are typically misleadingly high.

This paper has two objectives. The first is defining the nature of the difference in AG/SAG product size distributions as compared to conventional circuits, along with discussing operational factors that affect the size distribution. The second is proposing a method to analyse each portion of an AG/SAG-ball mill circuit separately to allow identification of the sources of grinding inefficiency.

**Influences on the Transfer Product Size Distribution**

The effect observed in the example in Figure 1 is typical, and illustrates the difficulty in describing the transfer particle size distribution by one point. To illustrate varying shapes of the size distribution curve without undue complication, we propose using a ratio between two points. For convenience, the ratio between the T80 and the T50 (the 80 and 50% passing particle sizes, respectively) is suitable. The ratio between these values (T80/T50) will be high for a wide size distribution (flat slope) and small for a tight size distribution (steep slope).

Figure 1. Comparison of the Particle Size Distribution Slopes

Figure 2 demonstrates the effect of the shape of the transfer size distribution on AG/SAG operating work indices. Finer transfer size distributions typically have steeper slopes (smaller T80/T50), and are more parallel to the final product size distribution (left graph). As a result, the operating work index calculated for the AG/SAG section (AWio) for finer transfer sizes is more representative of the work done by the AG/SAG mill (right graph). This figure is quite diagnostic of the problem discussed in this paper. It shows that the actual AG/SAG operating work index can be significantly biased, particularly for coarse transfer sizes. This result has led to misconceptions and terms such as “SAG mill inefficiency factor.” As ore breakage characteristics can be a major factor in AG/SAG mill product size distribution, Figure 2 compares only data for porphyry ores.

Before the true power efficiency of an AG/SAG grinding mill can be properly assessed by using the Wifi concept, the T50 used for calculations must be corrected to reflect the additional fines in the AG/SAG circuit product size distribution.

**Methods and Theory of 80 Corrections**

Essentially, the difficulty with using operating work indices for individual sections in AG/SAG circuits is that AG/SAG products contain more fines than their T80s indicate (based on a curve with the same T50 that is parallel to the product size distribution). In other words, AG/SAG mills generate substantial final product sized material that bypasses the ball mill. This is illustrated in Figure 3. The circuit flow rate is identified as Q, while the designations F, T and P represent the successive stage particle size distributions.

The Phantom Cyclone: This method is depicted in Figure 4, and consists of the insertion of a cyclone (identifiable as the phantom cyclone) between the AG/ SAG and the ball mill circuits (JKMRC, 1996). The phantom cyclone is used to classify the AG/SAG mill product into two products: the phantom cyclone overflow that is similar to the final ball mill product, and the phantom cyclone underflow is then used as an adjusted ball mill feed for the ball mill power calculation.

This technique is specifically suitable for model-based calculations, as it requires the simulation of a cyclone (or equivalent efficiency curve). The phantom cyclone model parameters are manipulated to ensure that the phantom cyclone overflow has a similar particle size distribution as the true cyclone overflow. The consequence is that the phantom cyclone underflow has a steeper particle size distribution that is roughly parallel to the true cyclone overflow. Figure 5 shows the product from the phantom cyclone and compares it to the transfer products. This method is sensitive to selected classification parameters, and there is no criterion for defining a parallel curve. Also, while the phantom cyclone is adequate for assessing power requirements for the ball mill section, it does not facilitate direct calculation of an AG/SAG circuit operating work index.

The Transfer Split: A.R. MacPherson Consultants Ltd. has developed an alternate method to correct for the transfer size effect. The method has similarities to the Phantom Cyclone technique, as it is also based on the decomposition of the transfer product. However, instead of being classified using the simulation of a cyclone, the transfer size distribution is mathematically decomposed into two products:

- Q1 is the percentage of material in the AG/SAG mill product that can be considered as final circuit product, or Fines Fraction. The Fines Factor is the Fines Fraction expressed as a percentage of circuit feed (Q1). T1’ is the 80% passing size of the Fines Fraction.
- Q2 is the fraction of material in the AG/SAG mill product that needs additional grinding in the ball mill, or “Corrected Ball Mill Feed.” T2’ is the 80% passing dimension of the Corrected Ball Mill Feed.

Q1 and Q2 are determined using the linear relationship between the transfer size distribution and the 80% passing size. Q1 and Q2 are defined by the ratio of the corrected ball mill feed (T2’) to the AG/SAG mill feed (T1').
The Fines Fraction corresponds to the fraction of the AG/SAG mill product that is fine enough to be considered “final product.” This fraction bypasses the ball mill, and should not be included in the ball mill feed mass. The fractions are considered separately as depicted in Figure 6.

Assuming that the particle size distribution T1’ must be identical to P (3) to be considered final product yields (4).

\[ T'_{1} = P \]  

(3)

\[ T'_{1} = P_{m} \]  

(4)

Then, the particle size distribution T2’ can be determined by a simple mass balance, i.e. by substituting a proportion of the final product (P) from the AG/SAG mill product size distribution (T) as follows:

\[ Q_{2} = Q - Q_{1} \]  

(5)

\[ T_{2}' = \left[ Q \times T - Q_{1} \times T_{1}' \right] / Q_{2} \]  

(6)

Finally, the corrected Bond operating work indices for the AG/SAG mill and ball mill can be re-organised as [7] and [8], where B’Wio and A’Wio are the corrected operating work indices and WAG/SAG, WBM are the respective power draw of AG/SAG and ball mills. Note that the ball mill operating work index is applied only on the portion that needs the additional grinding (Q2), which renders to the AG/SAG mill the credit for the fraction of the material that had been already ground to final product (Q1).

Figure 6. Schematic of the AG/SAG Mill PSA Decomposition

Figure 7. Variation of the T’2’-PSA for Variation in Fines Factor

Figure 8. Variation of Operating Work Indices with Fines Factor

To calculate the operating work indices, equations (5) and (6) must be solved. The equations contain three unknowns: 1. Q1: the Fines Factor, 2. Q2: the Corrected Ball Mill Feed, and 3. T2’: the particle size distribution of the Corrected Ball Mill Feed.

With two equations and three unknowns, there are several solutions to equations (5) and (6), depending on the value attributed to the Fines Factor (Q1), each yielding different Corrected Ball Mill Feed Size Distributions (T’2). This is depicted in Figure 7. Figure 8 presents the variation of the corrected operating work indices with Q1. It shows clearly the sensitivity of the Bond calculation to the amount of “final product” included in the feed to a grinding circuit.

With the addition of the constraint that states that the corrected ball mill feed (Q2) must have a cumulative passing curve (T’2) that is parallel to the ball mill product (P), a unique solution can be determined. The comparison of parallel curves would also allow for the utilisation of the Bond formula. Note that parallelism, which is defined as lines, which are “continuously equidistant,” is easier to apply on straight lines than on complex curves.

Usually, values of Q1 of 40 to 50% appear to make T2’ and P parallel. Between these values, the B’Wio is 12.9 to 15.1 kWh/t, compared to the uncorrected value of 8.6 kWh/t (Q1 = 0%). However, the determination of parallelism should rely on an objective criterion and not a subjective evaluation.

In order to quantify the parallelism of the curves, the use of the models proposed by Gaudin-Gates Shumann (GGS) and Rosin-Rammler Spirling (RRS) for particle size distributions was examined. These models are presented below:

\[ GGS: \quad \frac{W}{W_{o}} = \left( \frac{X}{X_{o}} \right)^{n} \]  

(9)

\[ RRS: \quad 1 - e^{-\left( \frac{T}{X_{o}} \right)^{m}} \]  

(10)

In these equations, W is the cumulative percent passing the dimension X. Xo represents the size of the largest particles in GGS and the 4962.2 for RRS. The parameter n (GGS) or m (RRS) both represent the slope of the curves in log-log or a Rosin-Rammer scale, respectively.

These models can be easily linearised and, in this form, can suit the definition of “continuously equidistant.” Both models were investigated and so far the GGS model has been found to provide more robust results than the RRS. In some circumstances, the GGS model cannot fit the entire AG/SAG mill particle size distribution, and in those cases the model had to be simplified. The logarithmic slope of the curves over a smaller part of the curve is sometimes more suitable. The fine transfer split results for our example are presented in Table 2. It shows that the transfer product can be decomposed into a 42% Fines Fraction that is identical to the ball mill product, with 58% of Adjusted Ball Mill Feed that needs additional grinding. The corrected ball mill operating work index using this method is 13.3 kWh/t, which is close to the Bond ball mill work index of the ore (13.4 kWh/t). The measured transfer size of 796μm produced by the AG/SAG mill corresponds to an equivalent 237μm (T’80), which can be back-calculated from the corrected ball mill operating work index by re-arranging the Bond formula [11].

Table 2. Correction Procedure Summary

<table>
<thead>
<tr>
<th>Fraction</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 (Fines Factor)</td>
<td>42%</td>
<td>58%</td>
</tr>
<tr>
<td>Q2</td>
<td>58%</td>
<td>58%</td>
</tr>
<tr>
<td>Equivalent T’80 (μm)</td>
<td>235</td>
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</table>

Table 3. Range of Case Study Operating Conditions

<table>
<thead>
<tr>
<th>PILOT PLANT DATA</th>
<th>SAG MILL</th>
<th>BALL MILL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed Rate (m³/hr)</td>
<td>0.933</td>
<td>0.933</td>
</tr>
<tr>
<td>WBM (μm)</td>
<td>171,000</td>
<td>726</td>
</tr>
<tr>
<td>WBM (μm)</td>
<td>726</td>
<td>24</td>
</tr>
<tr>
<td>Power (kW/hm)</td>
<td>21.2</td>
<td>6.31</td>
</tr>
<tr>
<td>Actual WiO (kW/hm)</td>
<td>28.5</td>
<td>8.6</td>
</tr>
<tr>
<td>Corrected WiO (kW/hm)</td>
<td>13.7</td>
<td>13.3</td>
</tr>
<tr>
<td>BWI (kW/hm)</td>
<td>235</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Table 3. Range of Case Study Operating Conditions

There are three major applications for this technique:

1. Existing grinding circuit analysis. Based on circuit survey samples, combined with throughput and power draws, the true operating work indices for both the AG/SAG mill and the ball mills can be calculated and compared to the AWi, RWi, or BWi, as appropriate. This comparison will show which units are operating efficiently and which are not.

2. Grinding circuit design. In a model-based design, the predicted transfer size can be split in a different manner as the phantom cycle. The model predicted power for the AG/SAG mill and the ball mill can then be compared to the power-based prediction.

3. Research and development. The knowledge of the true operating work index for the individual units of an AG/SAG mill circuit could benefit a variety of research and development projects focusing on increasing grinding efficiency.

For strictly power-based designs (without complimenting simulations), more work will be needed to develop a prediction of the transfer size slope based on the ore type and target operating conditions for the AG/SAG mill. The closing screen size and the recycle to the AG/SAG mill are significant factors. Useful relationships between grindability tests, mill size and operating conditions can be developed if the reliability of the method presented in this paper can be further developed.

CASE STUDIES
This section presents findings from case studies where the transfer split technique was applied. The case studies presented here are from four pilot plant investigations and three large plant surveys. The range of operating conditions studied is presented in Table 3.
Table 4 presents a summary of results for the transfer split procedure. The effect of the circulating load on AG/SAG and ball mill efficiency is presented in Figure 10. Test data collected with insufficient circulating loads were not included in the averages of Table 4. Insufficient circulating loads were defined as less than 200% for the ball mill and less than 30% for the AG/SAG mill. Note that when the AG/SAG work index was not available, the Bond rod mill was used for the calculations.

Table 4. Transfer Split Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{Wi} )</td>
<td>40%</td>
</tr>
<tr>
<td>( B'W_{0i}/B_{0i} )</td>
<td>75%</td>
</tr>
<tr>
<td>( B'W_{0i}/B_{0i} )</td>
<td>102%</td>
</tr>
<tr>
<td>( B'W_{0i}/B_{0i} )</td>
<td>138%</td>
</tr>
<tr>
<td>( A'W_{0i}/A_{0i} )</td>
<td>120%</td>
</tr>
<tr>
<td>( A'W_{0i}/A_{0i} )</td>
<td>34%</td>
</tr>
<tr>
<td>( A'W_{0i}/A_{0i} )</td>
<td>108%</td>
</tr>
<tr>
<td>( B'W_{0i}/B_{0i} )</td>
<td>57%</td>
</tr>
</tbody>
</table>

Of interest are the following:

1. The uncorrected transfer size is, on average, 2.5 times larger than the equivalent transfer size.
2. The uncorrected ball mill operating work index is, on average, 75% of the value of the BWi, while the corrected ball mill operating work index is, on average, similar to BWi.
3. The uncorrected AG/SAG mill operating work index is, on average, 170% of the small-scale AG/SAG work index. The correction brings this number, on average, to between the AG/SAG and the BWi.

A comparison between operating work indices and small-scale work indices before and after correction is made in Figure 11. Again AG/SAG indices are unfilled symbols, with solid symbols for ball mill indices. For tests with good overall grinding efficiency, the corrected operating work indices are typically close to the small-scale work indices.

Discrepancies between the corrected and the small-scale values can have two main sources. The first source is that inherent with the procedure and includes a lack of precision in the results obtained from the decomposition method or experimental errors in determining the small-scale or operating work indices. Also, the method assumes that Bond’s theory accurately judges the required grinding energy. While Bond’s equation is arguably more empirical than derived from first principles, it has proved amazingly robust in reliably predicting the energy required for grinding. As such, it provides an excellent if not абсолют benchmark.

The second source of discrepancy is associated with grinding inefficiencies in the operating plant. After correction, the BWi are generally quite close to the BWi, while the A’Wio was more often higher than the measured bench index. This corresponds to typical observations: given proper ball size selection and efficient classification, ball mills generally operate efficiently. A SAG circuit, however, has more variables involved in efficient operation (including appropriate steel charge, feed preparation, classification size, pebble crushing, and circuit power split). This method can identify which grinding section is inefficient. Collectively, the procedural sources of error are the bias without correction, and typically small enough to readily discern sources of grinding inefficiency.

Figure 12 compares corrected transfer sizes plotted against actual transfer sizes, and demonstrates that the actual T'80s produced by AG/SAG mills do not represent the real work performed by the mill in the context of Bond equations. The T'80 of an AG/SAG mill is, in fact, equivalent to a size that is 2 to 8 times (2.5 on average) finer with regards to ball mill grinding requirements. One grinding characteristics and the mill operation both influence the magnitude of this ratio.

CONCLUSIONS

This paper has demonstrated the strong influence that the slope of the size distribution of the AG/SAG mill product (or transfer size) has on the calculation of the operating work indices of the individual units in an AG/SAG/ball mill circuit. The fact that the AG/SAG mill product is not parallel to the ball mill final product introduces bias in those calculations. The work done by the ball mill, which benefits from the large amount of fines generated in the AG/SAG mill, is always overestimated. On the other hand, the AG/SAG mill always appears inefficient, and its true efficiency cannot be assessed from the T'80 without correction.

REFERENCES


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WHEN YOU NEED TO BE SURE